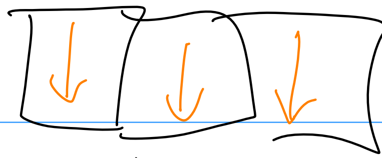
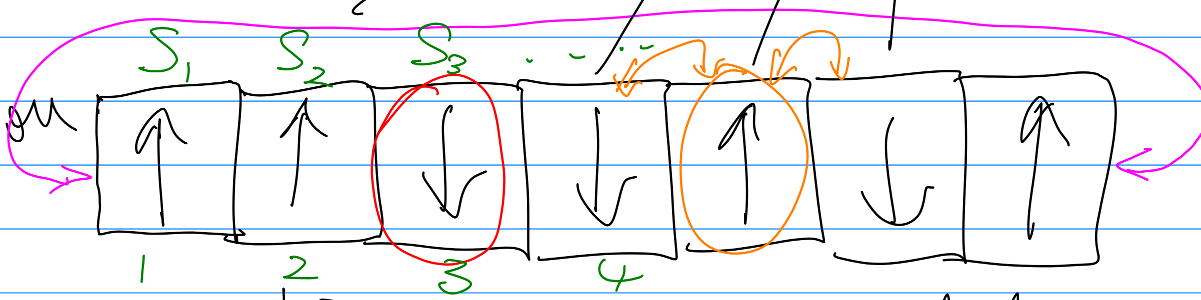
Ising Model



Potts Model

spin orientation 1D



1D Ising model

These models are subsets of CA models
but key feature = local interactions

Why are these useful? 1) Phase Transitions
2) Coarsening Problems

Interactions: simplify ⇒
low E ⇒ zero
high E ⇒ 1

Summation (w/ periodic b. conditions)

$$E = \frac{1}{2} \sum_{i=1,N} \sum_{j=1,NN} \left(1 - \delta_{S_i S_j} \right)$$

NN = set of nearest neighbors
N = cells

Minimize energy (coarsening)

Change a cell

Pick a cell at random

Calculate Energy $\left\{ \begin{array}{l} \text{initial state} \\ \text{~~proposed~~ new state} \end{array} \right.$

Complete system? NO

$E_{initial}$ vs. $E_{proposed}$

$$\Delta E = E_{proposed} - E_{initial}$$

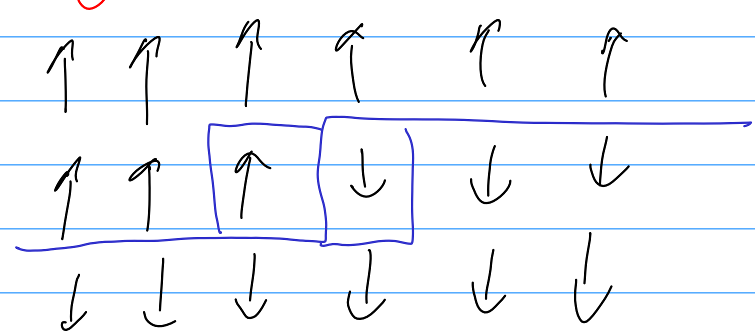
p = probability of accepting/making change

$$\begin{array}{ll} p=1 & < 0 \Rightarrow \text{accept change} \\ p=1 & = 0 \quad 50\% \;\; | \;\; \text{accept} \\ ? & > 0 \Rightarrow \text{don't do it} \end{array}$$

Typically in Potts model $\Delta E = 0$ is accepted

$\Downarrow$

random walk of defects on interfaces



Metropolis algorithm

positive ΔE? Finite "temperature"

not thermal temp.

$$\Delta E > 0 \qquad p = \exp^{-\Delta E / \boxed{kT}}$$

Uses of finite lattice temp.?

1) phase transitions
2) Interface roughness $\Rightarrow$ interface mobility

For any lattice and interaction Hamiltonian there is a critical lattice temp.

$T < T_{crit.} \Rightarrow$ single domain $(T=0)$

$T > T_{crit.} \Rightarrow$ disordered

Ferromagnetic: T_{curie}

Potts model?

Ising $\Rightarrow$ 0 or 1

Potts $\Rightarrow$ 0, 1, 2, 3 .... Q

$\uparrow \downarrow$ vs.

3D w/ 1st NN: tends to "pin" if operated at $T=0$

3D w/ 1st + 2nd + 3rd NN

Lattice Anisotropy: numerically overcome w/ finite temp.