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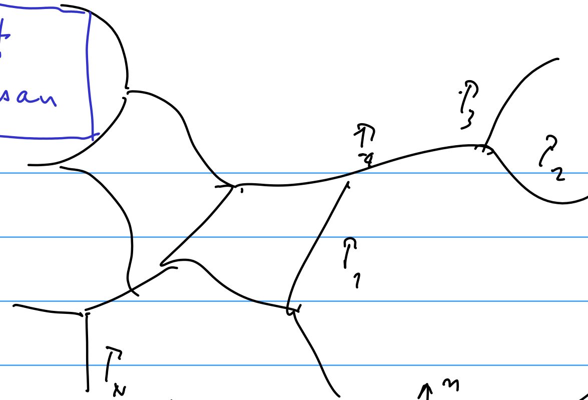
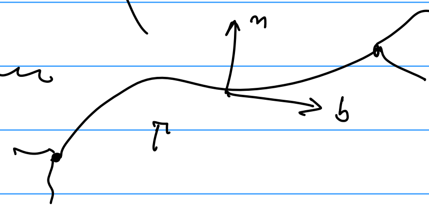
Evolving networks

metastable



$$v_n = \mu \frac{d}{ds} T \quad \text{on } \Gamma$$

$$\sum_{\partial\Gamma} T^{(i)} = 0 \quad \text{at } \phi \quad TJ \quad \text{Herring condition}$$



$$E = \sum_{\{\Gamma\}} \int_\Gamma \psi\, |b|\, ds \qquad \psi = \psi(n, \alpha)$$

$$\frac{dE}{dt} = \sum_{\{\Gamma\}} \int_\Gamma \left(\nabla_n \psi \cdot \frac{dn}{dt} |b| + \psi\, b \cdot \frac{db}{dt} \right) ds$$

evaluate at time t when we have chosen arc length parameter for all the curves

$$= \sum_{\{\Gamma\}} \int_\Gamma \left(R^T \nabla_n \psi + \psi\, b \right) \frac{db}{dt}\, ds$$

$$\frac{db}{dt} = \frac{\partial^2 \xi}{\partial t \partial s} = \frac{\partial}{\partial s} \frac{\partial \xi}{\partial t} = \frac{\partial}{\partial s} v$$

$$= \sum_{\{\Gamma\}} \int_\Gamma T \cdot \frac{dv}{ds}\, ds$$

$$= + \sum_{\{\Gamma\}} \left\{ - \int_\Gamma \frac{d}{ds} T \cdot v\, ds + v \cdot T \Big|_{\partial\Gamma} \right\}$$

$$v_n n = \mu \frac{d}{ds} T \qquad \mu > 0$$

$$= - \sum_{\{\Gamma\}} \int_\Gamma v_n^2 \cdot \frac{1}{\mu}\, ds + \boxed{\sum_{\{\Gamma\}} \sum_\Gamma v \cdot T = \sum v \cdot \sum_\Gamma T}$$

$= 0$ by Herring

$-$

?

$\Rightarrow$

$$\frac{dE}{dt} \leq 0 \qquad \text{dissipative for the energy}$$

$$\sum \int_0^\tau \int_\Gamma \frac{1}{\mu} v_n^2\, ds\, dt + E(\tau) = E(0)$$

"ensemble of spring mass dashpots"

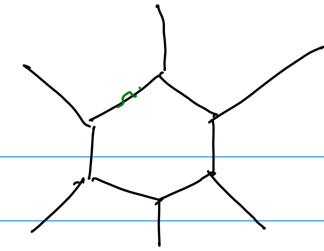
$E(t)$ decreasing (even when critical events are present)

2 kinds of results

1) local existence Bronsard + Reitich 1993

2) asymptotic stability

solve problem near stationary state for all time



Consider special case

$$v_n = \kappa$$

Γ: $x = \xi(x,t)$

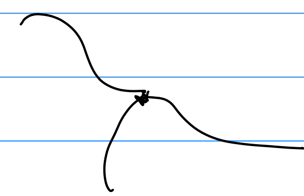
in $\mathbb{R}^2$ in $\mathbb{R}$

$$\frac{d\xi}{dt} = \kappa n + \mu_1 b \quad \Rightarrow \quad \frac{\partial \xi}{\partial t} = \kappa n + \mu b$$

maintain arclength parameter $\Longrightarrow$ reparametrize each time

$$\xi_x = |\xi_x| b \qquad \frac{db}{dx} = \kappa n \frac{dx}{ds} = \kappa n b$$

$$\xi_{xx} = |\xi_x|^2 \kappa n + |\xi_x|_x b$$



$$\frac{\partial \xi}{\partial t} = \frac{\xi_{xx}}{|\xi_x|^2} + \mu_1 b$$

$$\frac{\partial \xi}{\partial t} = \frac{\xi_{xx}}{|\xi_x|^2} \quad \longleftarrow \text{take this equation}$$

$$\frac{\partial u_1}{\partial t} = \frac{1}{(u_{1x}^2 + u_{2x}^2)} \frac{\partial^2 u_1}{\partial x^2} \qquad \frac{\partial u_2}{\partial t} = \frac{1}{(u_{1x}^2 + u_{2x}^2)} \frac{\partial^2 u_2}{\partial x^2}$$

$$(*) \quad \frac{\partial u_3}{\partial t} = \frac{1}{(u_{3x}^2 + u_{4x}^2)} \frac{\partial^2 u_3}{\partial x^2} \qquad \frac{\partial u_4}{\partial t} = \qquad 0 < x < 1,\ t > 0$$

$$\frac{\partial u_5}{\partial t} = \qquad \frac{\partial u_6}{\partial t} = \qquad \xi^{(1)} = (u_1, u_2),\ \text{etc}$$

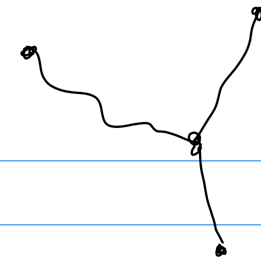
$$\sum \frac{\xi_x^{(r)}}{|\xi_x^{(r)}|} = 0 \quad \text{at } x = 0,\ t > 0$$

$$\xi^{(1)} = \xi^{(2)} = \xi^{(3)} \quad \text{at } x = 0,\ t > 0$$

$$\xi^{(r)} = \hat{\xi}^{(r)} \quad \text{at } x = 1,\ t > 0 \quad \text{(fixed)}$$

$$\sum_{j\,\text{odd}} \frac{u_{jx}}{\sqrt{u_{jx}^2 + u_{j+1x}^2}} = 0$$

Herring condition



$$\sum_{j\,\text{even}} \frac{u_{jx}}{\sqrt{u_{j+x}^2 + u_{jx}^2}} = 0$$

$u_1 = u_3 = \dots = u_5$, $u_2 = u_4 = u_6$ at $x = 0$

v = linear / assume for f_x $|v_x|^2 = 1$

$z = u - v$

$$\left(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right) u_1 = \left(\frac{1}{u_{1x}^2 + u_{2x}^2}\right) \frac{\partial^2 u_1}{\partial x_2}$$

$$\left(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right) z = \left(\frac{1}{u_{1x}^2 + u_{2x}^2} - 1\right) \frac{\partial^2 z}{\partial x^2} = f(z)$$

$$|f(z)| \leq C(z) \cdot |z_{xx}|$$

$$Bz = g(z, z_x)$$

$$z_{1x} + z_{3x} + z_{5x} = g_1$$

$$z_{2x} + z_{4x} + z_{6x} = g_2$$

Result: Can find some $\varepsilon > 0$ s.t. if

$$\int_\Omega z_{0xx}^2 \, dx \leq \varepsilon$$

then there is a smooth global solution of

$$z_t - z_{xx} = f(w, w_x, w_{xx}) \qquad t > 0$$

$$Bz = g(w, w_x)$$

$$z = z_0$$

will give us $z = Tw$ suitable w

which may then be iterated

$z = z_1 + z_2$

$$\begin{cases} \left(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right) z_1 = f\left(w, \frac{\partial w}{\partial x}, \frac{\partial^2 w}{\partial x^2}\right) & 0 < x < 1,\ t > 0 \\ \mathcal{B} z_1 = g\left(w, \frac{\partial w}{\partial x}\right) \\ z_1\big|_{t=0} = 0 \end{cases}$$

inhomogeneous equation
zero initial data

$$\begin{cases} \left(\frac{\partial}{\partial t} - \frac{\partial^2}{\partial x^2}\right) z_2 = 0 & 0 < x < 1,\ t > 0 \\ \mathcal{B} z_2 = 0 \\ z_2\big|_{t=0} = z_0 \end{cases}$$

homogeneous equation
given data