C S Smith (Euler characteristic)

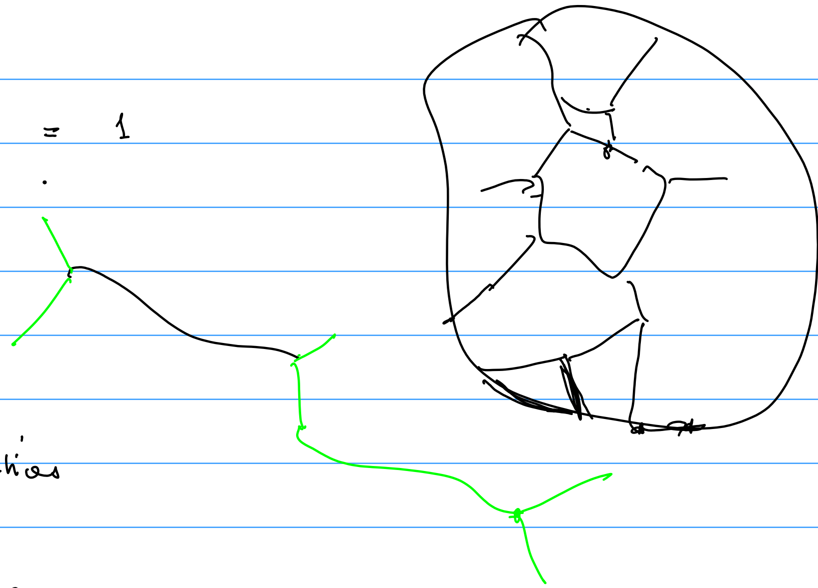
(i) $\nu_{\text{cells}} - \nu_{\text{edges}} + \nu_{\text{vertices}} = 1$

(ii) only TJ's present

each edge $\longleftrightarrow$ 2 vertices

each vertex $\longleftrightarrow$ 3 edges

$\Longrightarrow \quad 2\nu_{\text{edges}} = 3\nu_{\text{vertices}}$



(iii) cells and edges

: each edge belongs to 2 cells

ν_n = # cells with n edges

$$\frac{1}{2}\sum n\nu_n + \frac{1}{2}\nu_{\text{boundary}} = \nu_{\text{edges}}$$

Apply Euler

$$\sum \nu_n - \left(\frac{1}{2}\sum n\nu_n + \frac{1}{2}\nu_b\right) + \frac{2}{3}\left(\frac{1}{2}\sum n\nu_n + \frac{1}{2}\nu_b\right) = 1$$

$$\sum (6-n)\nu_n - \nu_b = 6$$

For a very large network

$$\sum (6-n)\nu_n \approx 0$$

f_n = fraction of cells with n edges : $\sum f_n = 1$

$$\sum (6-n)f_n = 0 \quad \text{or} \quad \sum n f_n = 6$$

expected value of # sides = 6

Graustein 1932 Annals of Math 32 149 - 153

What are the statistics of a configuration?

Lewis

f_n = fraction of n-sided cells

$$f_n = \alpha + \beta n$$

Rivier - Lissowski (1982)

$$\sum f_n = 1$$

$$\sum A_n f_n = A \qquad A_n = \text{average area of } n\text{-sided cell}$$

$$\sum (n-6) f_n = 0$$

$$\begin{pmatrix} 1 & \cdots & 1 \\ -3 & & N \\ A_3 & \cdots & A_N \end{pmatrix} \begin{pmatrix} f_3 \\ \vdots \\ f_N \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ A \end{pmatrix}$$

Suppose rank = 2

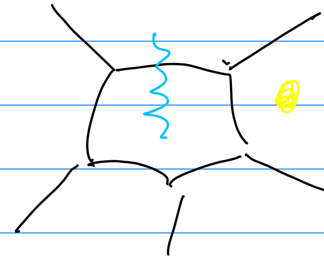
augmented last row = linear combination of augmented 1st + 2nd rows

$\Rightarrow$ this is the Lewis Law

a local law of evolution

b space filling constraints

c other features



interrogate to find larger scale statistics / distributions

simple examples ::

$\langle n \rangle = 6$ Euler rigorous

Aboav-Weaire empirical, can be surmised from evidence

$\langle A \rangle$ grows linearly empirical, do not have a good reason

Grain growth is a dissipative system

existence for a network

system of nonlinear parabolic PDE's

(1) local existence Bronsard + Reitich 1993

(2) asymptotic stability K + Lin 2001